Calc BC - Gold 3

① $y^3x + y^2x^2 = 6 \quad (2,1)$

$$3y^2\frac{dy}{dx}\cdot x + y^3 + 2yx^2\frac{dy}{dx} + 2xy^2 = 0$$

$$6\frac{dy}{dx} + 1 + 8\frac{dy}{dx} + 4 = 0$$

$$\frac{dy}{dx} = \frac{-5}{14} \quad C$$

② $f(x) = \sin^2(3-x)$

$$f'(x) = 2\sin(3-x)\cdot\cos(3-x)(-1)$$

$$f'(0) = -2\sin(3)\cdot\cos(3) \quad B$$

③ work backwards I and II C

④ $y = x + \sin(xy)$

$$\frac{dy}{dx} = 1 + \cos(xy)\left(y + x\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = 1 + y\cdot\cos(xy) + x\cdot\cos(xy)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 + y\cdot\cos(xy)}{1 - x\cdot\cos(xy)} \quad E$$

(5) $2xy^2 - 3y = 18 \quad (3,2)$

$2y^2 + 4xy\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$8 + 24\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-8}{21}$ slope of tangent

slope normal $= \frac{21}{8}$ (A)

(6) $f(x) = \cos(3\pi x^2)$

$f'(x) = -6\pi x \cdot \sin(3\pi x^2)$

$f'(\frac{1}{3}) = -2\pi \sin(\frac{\pi}{3}) = -2\pi \cdot \frac{\sqrt{3}}{2}$

$= -\pi\sqrt{3}$ (A)

(7) $f(x) = \tan^{-1}(x)$

$f'(x) = \frac{1}{1+x^2} \quad f'(2) = \frac{1}{5}$ (B)

(8) $f'(x) = 3x^2 - 4x - 4 \cdot e^{x^3 - 2x^2 - 4x + 5}$, $f'(x) = 0$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$x = -\frac{2}{3}, x = 2$

$3x+2$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
	$\nearrow$ $-\frac{2}{3}$	$\searrow$ 2	$\nearrow$

min at $x = 2$ (E)

(9) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-5}{3t^2}$ $10 = t^3 + 2$

$8 = t^3$

$t = 2$

$\left.\frac{dy}{dx}\right|_{t=2} = -\frac{1}{12}$ (D)

(10) $x(t) = 4t^2 - \sin 3t$

$v(t) = 8t - 3\cos 3t$

$a(t) = 8 + 9\sin 3t$

$a(\frac{\pi}{2}) = 8 + 9\sin(\frac{3\pi}{2})$

$= 8 - 9 = -1$ (A)

(11) $e^{xy} + 4y = 7$

$e^{xy}(y + x\frac{dy}{dx}) + 4\frac{dy}{dx} = 0$

$y \cdot e^{xy} + x \cdot e^{xy}\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-ye^{xy}}{x \cdot e^{xy} + 4}$ (E)

(12) $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$dV = 4\pi(3)^2(.1)$

≈ 11.3 (A)

(13) $x = 2t$, $t = \frac{1}{2}x$

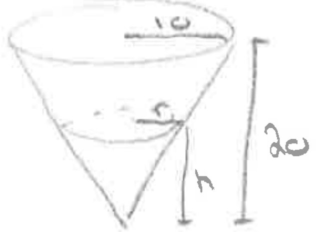
$y = 4t - t^2$

$= 4(\frac{1}{2}x) - (\frac{1}{2}x)^2$

$= 2x - \frac{1}{4}x^2$

Parabola (E)

(14)



$V = \frac{1}{3}\pi r^2 h$ $\qquad \frac{dh}{dt} = \frac{1}{2}$

$\frac{r}{h} = \frac{10}{20}$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi(\frac{1}{2}h)^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi (8)^2 (\frac{1}{2})$

$= 8\pi$ (B)

(15) $e^{f(x)} = 1 + x^2$

$f(x) = \ln(1 + x^2)$

$f'(x) = \frac{2x}{1+x^2}$ (B)

(16) $f'(c) = \frac{f(b) - f(a)}{b - a}$

$3c^2 - 4c = \frac{f(2) - f(0)}{2}$

$3c^2 - 4c = 0$

$c(3c - 4) = 0$

$c = 0, \; c = \frac{4}{3}$ (D)

Note: $c \in (0, 2)$

(17) $g'(-2) = \frac{1}{f'(5)}$

$= \frac{1}{-\frac{1}{2}}$

$= -2$ (E)

(18) $\lim_{x \to \infty} \frac{2x^2 + 1}{4 - x^2} = -2$

(B)

(19) $A = \pi r^2$ $\qquad 64\pi = \pi r^2$

$8 = r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$64\pi = 2\pi(8)\frac{dr}{dt}$

$\frac{dr}{dt} = 6$ (A)

(20) (E)

(D is not continuous)

(21) (B)

(22) concave down

$\frac{f(1.2)-f(1.1)}{1.2-1.1} < f'(1.2) < \frac{f(1.3)-f(1.2)}{1.3-1.2}$

$1.8 < f'(1.2) < 2.0$

(D)

(23) $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$

want $g'(t)$ most negative

graph $g'(t) = 20 \cdot \frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right) - 10 \cdot \frac{\pi}{6}\sin\left(\frac{\pi t}{6}\right)$ on $[0, 8]$

$= 10\pi\cos\frac{\pi t}{2} - \frac{5}{3}\pi\sin\left(\frac{\pi t}{6}\right)$

$t = 2.017$ (B)

(24) work backwards by differentiating each (Please don't bother w/ A & B)

C

(25) II only B

I. left ≠ right

III. deriv does not exist at $x=1$

(26) $x_1(t) = 2t^2 - 5t + 7$ $x_2(t) = \sin 2t$

$v_1(t) = 4t - 5$ $v_2(t) = 2\cos(2t)$

$a_1(t) = 4$ $a_2(t) = -4\sin(2t)$

$4 = -4\sin(2t)$

Graph $4 + 4\sin(2t) = 0$ look for Zeros

B

(27) $f(x) = e^x - x^3$

$f'(x) = e^x - 3x^2$

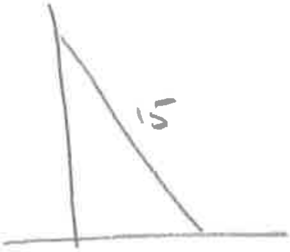
graph $f'(x)$ and look for Zeros

D

(28)

A

(29)



$x^2+y^2=15$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

when $x=9,\ y=12$

$2(9)\frac{dx}{dt}+2(12)(-12)$

$\frac{dx}{dt}=16$ ft/sec

D

(30) $A=2xe^{-x^2}$ graph to find maximum

D $x=.707,\ y=.858$

(31) $f'(x)=x(x-1)^2(x-2)^3(x-3)^4$

x	$-$	$+$	$+$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$	$+$	$+$
$(x-2)^3$	$-$	$-$	$-$	$+$	$+$
$(x-3)^4$	$+$	$+$	$+$	$+$	$+$
	$+$	$-$	$-$	$+$	$+$
	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$

0, 1, 2, 3

B

(32) $x^3+xy-y^2=10$

$3x^2+y+x\frac{dy}{dx}-2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-(3x^2+y)}{x-2y}$

vertical tangent $\Rightarrow x-2y=0$

$y=\frac{1}{2}x$

$x^3+x(\frac{1}{2}x)-(\frac{1}{2}x)^2=10$

$x^3+\frac{1}{2}x^2-\frac{1}{4}x^2=10$

$x^3+\frac{1}{4}x^2-10=0$

graph look for zeros.

C

(33) i) look for sign change on graph

(C)

ii) recall: speed $= |v(t)|$ (E)

(34) $x_1(t) = \cos(t^2+1)$

$v_1(t) = -\sin(t^2+1)\cdot 2t$

$x_2(t) = \dfrac{e^t}{2t}$

$v_2(t) = \dfrac{2t\cdot e^t - 2e^t}{4t^2}$

graph both, look for intersection,

(E)

(35) x-intercept, $y=0$

$0 = t - \dfrac{t^3}{9}$

$t^3 - 9t = 0$

$t(t+3)(t-3) = 0$

$t=0,\ t=\pm 3$ smallest $t=-3$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-\frac{1}{3}t^2}{e^t}$$

$$\left.\frac{dy}{dx}\right|_{t=-3} = -40.171$$ (A)

(36) a-d, f-h look at solutions

e. $y-2=-(x-6)$

$y=-x+8$

(37) $1350e^{.07606t}=50,000$

$e^{.07606t}=\frac{5,000}{135}$

$t=\frac{\ln\left(\frac{5000}{135}\right)}{.07606}\approx 47.488$

$1960+47\approx 2007$

(38) Look at graph

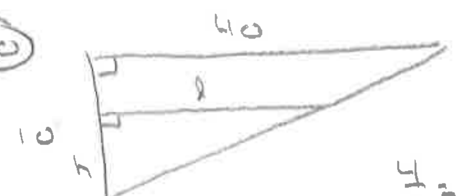
(39) $f'(x)=e^{\sin x}$, $f(0)=1$

$f'(0)=e^{0}=1$

$L(x)=1+1(x-0)=x+1$

$f(.25)\approx L(.25)=1.25$

(40)



$\frac{4}{1}=\frac{l}{h}$

$l=4h$

a) $V=\frac{1}{2}l\cdot w\cdot h$

$V=10lh$

$V=10(4h)h=40h^2$

$\frac{dV}{dt}=80h\frac{dh}{dt}$

$16=80\sqrt{10}\cdot\frac{dh}{dt}$

when $V=400$

$400=40h^2$

$100=h^2$

$h=\sqrt{10}$

$\frac{dh}{dt}=.0632$ ft/min

b) $V=\frac{1}{2}(40)(20)(10)$

$=4000$ ft^3

$t=\frac{4000}{16}=250$ min

≈ 4.165 hrs

c) $\frac{dh}{dt}=\frac{16}{80h}$

$\approx .022$ ft/min